MOUNTAIN TORQUE AND EXTERNAL FORCING

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ABSTRACT

Theoretical research and numerical modelling show that, although mountain torque owes its existence to the unevenness of the ground surface, its sign and intensity depend strongly upon the relative disposition between mechanical and thermal forcing. The strong annual variations of mountain torque in the northern subtropics revealed by Yeh and Zhu (1958) are attributed to the different thermal features of Tibetan Plateau between winter and summer.

INTRODUCTION

In addition to frictional torque, mountain torque is another kind of surface source of atmospheric angular momentum. According to Eliassen-Palm (EP) theorem, a steady, conservative system possesses $\nabla \cdot \boldsymbol{F}=0$, where $\boldsymbol{F}$ stands for EP flux. In other words, the absence of heat and angular momentum source corresponds to zero cross-latitude eddy transfer of potential vorticity. Thus, mountain torque, together with frictional torque, plays significant roles in atmospheric angular momentum budgets. In a quasi-geostrophic framework, it can be shown that both mechanical forcing and thermal forcing have strong impacts on mountain torque, although this torque cannot exist if there is no mountain.

I. MECHANICAL FORCING

At steady state, the Charney-Drazin (1961) system can be expressed as

$$\bar{u}\frac{\partial}{\partial x}\nabla^2\psi+\beta\frac{\partial\psi}{\partial x}=f_0\left(\frac{\partial}{\partial z}-\frac{1}{H}\right)w, \tag{1}$$

$$\bar{u}\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial z}\right)+\frac{N^2}{f_0}w=0. \tag{2}$$

With

$$n^2=\frac{N^2}{f_0^2}[\beta/\bar{u}-(k^2+l^2)]-\frac{1}{4H^2} \tag{3}$$

and a lower boundary condition

$$w=\bar{u}_0\frac{\partial h}{\partial x} \qquad \text{at} \quad Z=0, \tag{4}$$

where h denotes mountain height

$$h=Re[h_m\cos ly\exp i(kx+\lambda_0)], \tag{5}$$

λ_0 stands for phase difference between orography and stream function, one can obtain (Wu, 1984) the following results:

(a) For short waves, $K^2+l^2>\beta/\bar{u}-f_0^2/4H^2N^2$, solution is evanescent in the vertical

$$\psi(x,y,z)=\left[\frac{\rho(0)}{\rho(z)}\right]^{1/2}N^2f_0^{-1}[|n|-(2H)^{-1}]^{-1}h\exp(-|n|z). \quad (6)$$

There are no upward propagation of wave energy (trapped) and poleward transfer of enthalpy, nor is the mountain torque.

(b) For planetary waves, CD criterion is satisfied, i.e.

$$0<\bar{u}<\bar{u}_c=\beta(K^2+l^2+f_0^2/4H^2N^2)^{-1}. \quad (7)$$

Upon the adoption of the Boussinesq approximation, the following solution can be reached:

$$\psi(x,y,z)=-\left[\frac{\rho(0)}{\rho(z)}\right]^{1/2}\frac{Nh_m\cos\lambda_0}{[\beta/\bar{u}-(k^2+l^2)]^{1/2}}\cos ly\sin(kx+nz) \quad (8)$$

which gives surface pressure perturbation

$$\Delta P^m=\bar{\rho}^2f_0^2\,\bar{u}h_m\beta^{-1}H^{-1}\cos\lambda_{0m}\simeq 5h_m \qquad \text{(hpa)} \quad (9)$$

with h_m in units of kilometre.

For a zonal belt with width $\Delta\varphi$, mountain torque is defined as

$$T_M=2\pi a^3\cos^2\varphi f_0\rho_0\overline{v_0^*h^*}\Delta\varphi. \quad (10)$$

From (5) and (8), in the case of mechanical forcing

$$\begin{aligned}T_M^m&=-\alpha_m\cos^2\varphi\cos^2(la\varphi)\Delta\varphi,\\ \alpha_m&=\pi a^3f_0\rho_0Nk[\beta/\bar{u}-(k^2+l^2)]^{-1/2}h_m^2\cos\lambda_{0m}, \qquad k^2+l^2<\beta/\bar{u}\end{aligned} \quad (11)$$

II. THERMAL FORCING

Assuming that vertical heating scale H_Q satisfies $(H_Q/6\text{ km})^2\ll 1$, and planetary scale motions are nearly horizontal. The thermal dynamic equation can then be linearized to

$$\bar{u}\frac{\partial T}{\partial x}\simeq Q=Re[Q_T\exp(ikx)].$$

The omission of the term $v\,\partial\bar{T}/\partial y$ will not violate the following discussions at least qualitatively, provided that $\bar{u}\gg v$, and H_Q is small. Thus

$$\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)=\frac{gQ}{T\bar{u}}. \quad (12)$$

At a level ($Z\approx H/2$) where vertical motion is maximum,

$$v=\frac{1}{f_0\rho}\frac{\partial p}{\partial x}\simeq\frac{f_0}{\beta}\frac{\partial w}{\partial z}=0.$$

Integration of (12) from this level downwards to the surface gives

$$\frac{\partial p_0}{\partial x}=-\frac{\rho_0gH}{2\bar{T}\bar{u}}Q,$$

where $\bar{T}$ and $\bar{u}$ are quantities averaged over the lower troposphere. This gives surface pressure perturbation

$$\Delta P^T = -\frac{\rho_0 g H L}{2\pi \overline{T}\,\overline{u}} Q_T \simeq -28 Q_T \quad \text{(hpa)} \tag{13}$$

with Q_T in units of K day^{-1}. Obviously, under normal conditions (17), ΔP^T is about 3 times as large as ΔP^m (see (9)). Now, we obtain

$$\overline{v^* h^*} = -\frac{gH}{2 f_0 \overline{T}\,\overline{u}} \overline{Q^* h^*} \quad . \tag{14}$$

Let

$$Q = \overline{Q} + Q_T \cos kx \cos la\varphi, \tag{15}$$

(5), (10), (14) and (15) then lead to

$$\boxed{\begin{aligned} T_M^T &= -\alpha_T \cos^2\varphi \cos^2(la\varphi)\,\Delta\varphi \\ \alpha_T &= \pi a^3 \rho_0 g H (2\overline{T}\,\overline{u})^{-1} Q_T h_m \cos\lambda_{0T} \end{aligned}} \tag{16}$$

III. COMPARISON OF MECHANICAL FORCING AND THERMAL FORCING

Firstly, we notice $\alpha_m > 0$, so $T_M^m < 0$ (see (11)). Therefore, mountain torque T_M^m due to mechanical forcing solely must be negative. This is true since orography-induced anticyclone is located to the west of mountain ridge. On the other hand, the sign of α_T in (16) depends on the correlation between Q_T and h_m. Thus, if mountain coincides with heat sink (source), α_T is negative (positive), and mountain torque T_M^T due to thermal forcing must be positive (negative).

Secondly, take the following normal values:

$$\begin{array}{lll} a = 6.4\times10^6\text{m}, & f_0 = 1\times10^{-4}\text{s}^{-1}, & N = 10^{-2}\text{s}^{-1} \\ \rho_0 = 1.3\text{kg m}^{-3}, & \beta = 1.5\times10^{-11}\text{m}^{-1}\text{s}^{-1}, & \overline{u} = 8\ \text{ms}^{-1} \\ k = l = 2/a, & h_m = 1\text{km}, & g = 10\text{ms}^{-2} \\ H = 10^4\text{km}, & \overline{T} = 265\text{K}, & Q_T = 1\text{K d}^{-1}, \\ \cos\lambda_0 = 0.5, & \Delta\varphi = 5^0 & \end{array} \tag{17}$$

at $\varphi = 30^0$, mountain torque T_M^m and T_M^T are evaluated to be

$$\begin{cases} |T_M^m| \approx 2 \text{ Hadley}, \\ |T_M^T| \approx 5.6\text{Hadley}, \ 1 \text{ Hadley} = 10^{18}\text{kgm}^2\text{s}^{-2}. \end{cases} \tag{18}$$

Despite the linear distortion, (18) does show us that mountain torque due to thermal forcing is much stronger than that due to mechanical forcing. Under normal atmospheric conditions, the ratio between the two

$$\left|\frac{T_M^T}{T_M^m}\right| = \frac{gH}{2f_0\overline{T}\,\overline{u}N} \frac{[\beta/\overline{u} - (k^2+l^2)]^{1/2}}{k} \left|\frac{Q_T}{h_m}\right| \frac{\cos\lambda_{0T}}{\cos\lambda_{0m}} \tag{19}$$

is about 3.

IV. NUMERICAL MODELLING

Dividing a model atmosphere into two layers as shown in Fig. 1, a quasi-geostrophic baroclinic long wave model can be constructed by writing the vorticity equation at odd levels and thermodynamic equation at the interface (Wu, 1984):

$$\begin{cases} \frac{D}{Dt_3}(\zeta_3+f) = -\frac{2f_0}{H}w_2 \\ \frac{D}{Dt_1}(\zeta_1+f) = \frac{2f_0}{H}(w_2-w_0) \\ \frac{D}{Dt_2}\psi_s + \frac{HN^2}{2f_0}w_2 = -\gamma(\psi_s-\tilde{\psi}_s), \end{cases} \tag{20}$$

where $\psi_s=\psi_3-\psi_1$ represents the thickness of the model atmosphere, and the substantial derivative

$$\frac{D}{Dt_i}=\frac{\partial}{\partial t}+\boldsymbol{V}_i\cdot\nabla \quad (i=\overline{1,3}). \tag{21}$$

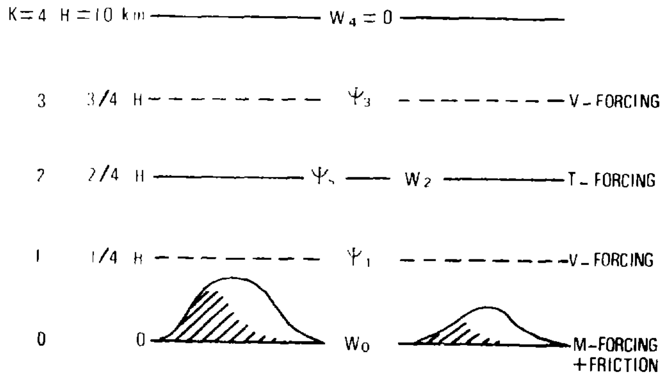

Fig. 1. Vertical resolution of the model atmosphere. Mechanical forcing is applied on the surface; thermal forcing at the interface between two levels.

Boundary vertical motion w_0 is considered to result from the unevenness of the surface and the pumping and/or suction of the Ekman layer, i.e.

$$w_0=\boldsymbol{V}_0\cdot\nabla h+\alpha\nabla^2\psi_0, \quad \text{with} \quad \alpha=(K/2f_0)^{1/2}, \tag{22}$$

where K is the eddy viscosity coefficient, taken as $10\,m^2s^{-1}$; (20) may be transfered to a nondimensional system, and, upon employing normalized associated Legendre polynomials, to a set of spectrum coefficient tendency equations. Time integration and numerical experiment can therefore be based on Wu (1987). External mechanical and thermal forcing are introduced by specifying a priori the coefficients of the following forcing functions:

$$\begin{cases} \eta=\frac{f_0}{\Omega H}h=\sum_n\sum_m(hA_n^m\cos m\lambda+hB_n^m\sin m\lambda)P_n^m(\sin\varphi) \\ S=\frac{\Omega}{a^2}\tilde{\psi}_s=\sum_n\sum_m(SA_n^m\cos m\lambda+SB_n^m\sin m\lambda)P_n^m(\sin\varphi) \end{cases} \tag{23}$$

Two sets of experiment, M and TM were designed to demonstrate solely mechanical forcing and combined mechanical and thermal forcing respectively. For the M case,

$$\begin{cases} SA_2^0=-30\text{K}/Ts & SA_4^0=7\text{K}/Ts \\ HA_3^1=-2000\text{ m}/Hs & HB_2^1=1000\text{ m}/Hs \\ SA_n^m=SB_n^m=0 & \text{if } m\neq 0, \end{cases} \tag{24}$$

where $H_s=\Omega Hf_0^{-1}=7854$ m is a constant height measure,

$T_s = 2f_0\Omega a^2 R^{-1} = 1750\text{K}$ is a constant temperature measure. For the TM case,

$$\begin{cases} SA_2^0 = -22\text{K}/Ts, & SA_1^0 = 3\text{K}/Ts, \\ SA_2^1 = -10\text{K}/Ts, & SB_2^1 = -15\text{K}/Ts, \\ SA_3^2 = 37\text{K}/Ts, & SA_5^2 = 5\text{K}/Ts, \\ hA_3^2 = -2000\text{m}/Hs, & hB_2^1 = 1000\text{m}/Hs. \end{cases} \tag{25}$$

This gives about 1K/day cooling over two mountain ridges and heating over two "oceans". In steady state, the latitudinal distributions of mountain torque, frictional torque and surface zone wind for cases M and TM are shown in Fig. 2 (a) and (b) respectively.

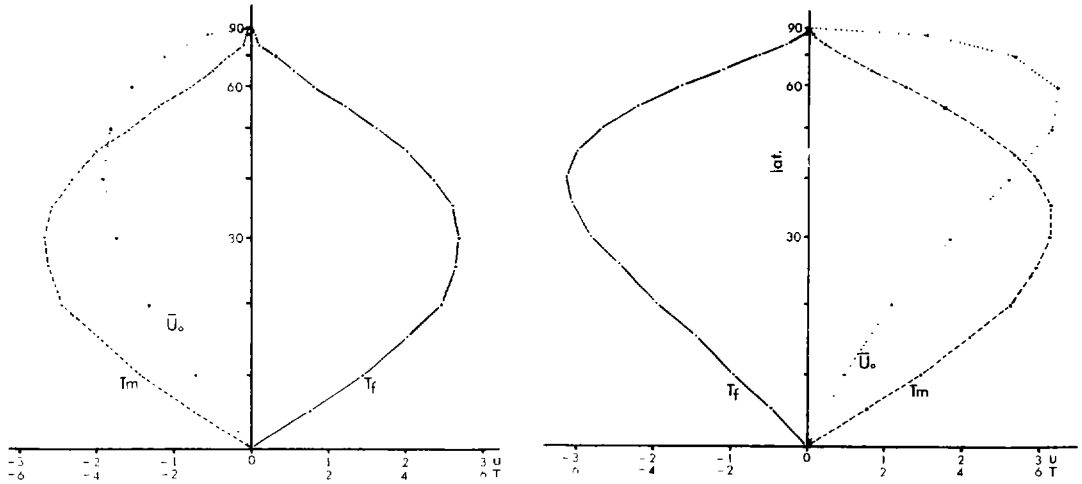

Fig. 2. The latitudinal distributions of mountain torque T_m, frictional torque T_f and zonal mean zonal surface wind U_0, with $HA_1^2 = -2000m/Hs$, $HB_2^1 = 1000m/Hs$. U_0 in units of ms^{-1}, T in Hadley. (a) M-forcing case, $SA_n^m = SB_n^m = 0$, if $m \neq 0$; (b) TM-forcing case, referred to (25). Adopted from Figs. 6. 1 and 6.2 of Wu (1984).

In the M case, T_m is negative as shown in (11). On the contrary, it is positive in the TM case. This is due to the fact that T_M^T in (16) depends on the sign of product $Q_T hm$. According to (25),

$$Q_T hm \propto SA_3^2 hA_3^2 + SB_2^1 hB_2^1 < 0,$$

i.e., mountains coincide with cooling sources. Thus surface high pressure centre appears to the east of mountain ridge. Therefore, mountain torque acts to accelerate westerlies. It is also interesting to consider their magnitudes. At 30°N, $T_M^m = -5.4$ Hadley. If we approximately regard the torque in case TM to be the sum of T_M^m and T_M^T (it is not actual, see Wu, 1984), then $T_M^T + T_M^m = 6.3$ Hadley at the same latitude. Thus $T_M^T = 11.7$ Hadley $\gg |T_M^m|$, somewhat smaller than expected from (19). This is due to the fact that linearity was employed to purchase (19), and, probably more importantly, that corresponding to a fixed zonal wind $\bar{u}$ and diabatic heating Q, the omission of the term $v\,\partial\bar{T}/\partial y$ requires a stronger temperature contrast $\partial T/\partial x$, which results in stronger surface pressure perturbation in (13) as well as mountain torque in thermal forcing case. Despite this quantitative discrepancy, the general agreement between theory and modelling results is encouraging.

V. DATA ANALYSES

In real atmosphere, Yeh and Zhu (1958) showed that the distribution of mountain torque in subtropical latitudes, where the main body of the Tibetan Plateau is located, changes sign from winter to summer, in strong contrast to its constancy in other latitude belts as shown

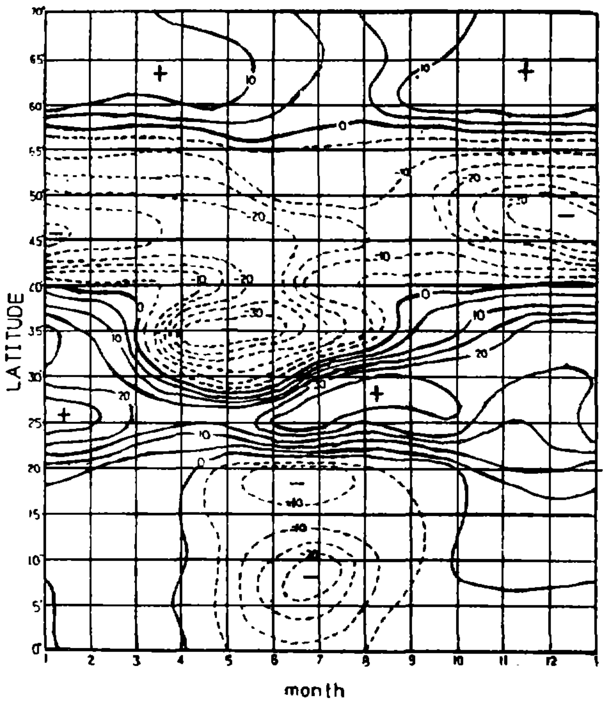

Fig. 3. Annual variation of mountain torque exerted upon the atmosphere at different latitudes. Units of 0.1 Hadley. Ordinate: latitudes; abscissa: month in the order Jan. to Jan. Copied from Fig. 8.1 of Yeh and Zhu (1958).

in Fig. 3. Investigations of the distribution of diabatic heating of the atmosphere, such as the indirect calculations of Aubert and Winston (1951). Zhu (1961), staff Members, Academia Sinica (1958), Kubota (1970), Asakura (1970); and the direct calculations of Clapp (1961) and Manabe (1964), etc., all show that there exist heat sources in winter and sinks in summer over the Northern Hemispheric oceans. On the other hand, over the Tibetan Plateau, there exist heat sink in winter and source in summer. This results in substantial heat contrast along northern subtropical latitudes. In the east of Tibetan Plateau, strong surface high appears in winter, whereas strong low is observed in summer. It is this thermal effect that results in the prominent annual variation of subtropical mountain torque. Therefore, through exerting its mountain torque upon the atmosphere, the Tibetan Plateau plays the roles of accelerating atmospheric westerlies in winter and decelerating westerlies in summer.

VI. CONCLUSIONS

In purely mechanical forcing, when westerlies impinge upon orography, although their kinetic energy is conserved, their angular momentum losses substantially due to dynamic effects. However, if the effects of thermal forcing are taken into account, the gain or loss of westerly angular momentum of the atmosphere, when interacting with orography, depends on the relative disposition of mechanical and thermal forcing. The magnitude of mountain torque due to thermal forcing is much stronger than that due to mechanical forcing. Therefore the observed latitudinal distirbution of mountain torque must depend substantially on thermal forcing.

Over the Tibetan Plateau, there exist heat sources in summer and sinks in winter. These are closely relevant to the downstream high in winter and the low in summer. Such combinations of mechanical forcing and the thermal forcing of Tibetan Plateau contribute to the unique annual variations of mountain torque in northern subtropical latitudes, and to the acceleration in winter and deceleration in summer of atmospheric westerlies.

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